NS8-24 Multiplying Fractions by Whole Numbers

REMINDER ▶ Multiplication is a short form for addition.

$3 \times 4 = 4 + 4 + 4$ $\quad$ $5 \times 7 = 7 + 7 + 7 + 7 + 7$ $\quad$ $2 \times 9 = 9 + 9$

1. Write each product as a sum.

   a) $3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\quad$ b) $2 \times \frac{3}{7} = \frac{3}{7} + \frac{3}{7}$ $\quad$ c) $4 \times \frac{5}{11} = \frac{5}{11} + \frac{5}{11} + \frac{5}{11} + \frac{5}{11}$

2. Write each sum as a product.

   a) $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \times \frac{1}{2}$ $\quad$ b) $\frac{5}{9} + \frac{5}{9} = 2 \times \frac{5}{9}$ $\quad$ c) $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 5 \times \frac{3}{4}$

REMINDER ▶ To add fractions with the same denominator, add the numerators.

3. Find the products by first writing each product as a sum.

   a) $4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$
   $= \frac{12}{5} = 2\frac{2}{5}$

   b) $2 \times \frac{3}{4} = \frac{3}{4} + \frac{3}{4}$
   $= \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

   c) $2 \times \frac{4}{7} = \frac{4}{7} + \frac{4}{7}$
   $= \frac{8}{7} = 1\frac{1}{7}$

   d) $5 \times \frac{4}{11} = \frac{4}{11} + \frac{4}{11} + \frac{4}{11} + \frac{4}{11} + \frac{4}{11}$
   $= \frac{20}{11} = 1\frac{9}{11}$

   e) $6 \times \frac{3}{7} = \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7} + \frac{3}{7}$
   $= \frac{18}{7} = 2\frac{4}{7}$

To multiply a fraction with a whole number, multiply the numerator by the whole number and leave the denominator the same.

Example: $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{2+2+2}{9}$ so $3 \times \frac{2}{9} = \frac{3 \times 2}{9}$

4. Multiply the fractions with the whole number. Write your answer as a mixed number.

   a) $4 \times \frac{3}{7} = \frac{4 \times 3}{7} = \frac{12}{7} = 1\frac{5}{7}$

   b) $5 \times \frac{2}{3} = \frac{5 \times 2}{3} = \frac{10}{3} = 3\frac{1}{3}$

   c) $3 \times \frac{4}{5} = \frac{3 \times 4}{5} = \frac{12}{5} = 2\frac{2}{5}$

5. Find the products. Simplify your answer.

   a) $3 \times \frac{4}{6} = \frac{12}{6} = 2$

   b) $8 \times \frac{3}{4} = \frac{24}{4} = 6$

   c) $5 \times \frac{4}{10} = \frac{20}{10} = 2$

   d) $3 \times \frac{6}{9} = \frac{18}{9} = 2$

   e) $12 \times \frac{2}{8}$ $2\frac{24}{8} = 3$

6. Find the products.

   a) $4 \times \frac{5}{4} = \frac{20}{4} = 5$

   b) $3 \times \frac{2}{3} = \frac{6}{3} = 2$

   c) $7 \times \frac{9}{7} = \frac{63}{7} = 9$

   d) $8 \times \frac{5}{8} = \frac{40}{8} = 5$

   e) $a \times \frac{b}{a} = \frac{ab}{a} = b$

In mathematics, the word "of" can mean multiply.

Examples: "2 groups of 3" means 2×3

"6 groups of $\frac{1}{2}$" means $6 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

"$\frac{1}{2}$ of 6" means $\frac{1}{2} \times 6$ Reminder: $\frac{a}{b}$ of c is $a \times c \div b$

7. Calculate each product by finding the fraction of the whole number.

a) $\frac{1}{3}$ of 6 = 2 so $\frac{1}{3} \times 6 =$ 2

b) $\frac{3}{5}$ of 10 = 6 so $\frac{3}{5} \times 10 =$ 6

c) $\frac{2}{3}$ of 6 = 4 so $\frac{2}{3} \times 6 =$ 4

d) $\frac{3}{4}$ of 20 = 15 so $\frac{3}{4} \times 20 =$ 15

When multiplying whole numbers, the order we multiply in does not affect the answer.

Examples: $2 \times 3 = 3 \times 2 = 6$ $4 \times 5 = 5 \times 4 = 20$

INVESTIGATION 1 ▶ When multiplying a fraction and a whole number, does the order we multiply in affect the answer?

A. Calculate the products in both orders.

i) $8 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{8}{4} = 2$

$\frac{1}{4} \times 8 = \frac{1}{4}$ of 8 = 2

ii) $6 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{12}{3} = 4$

$\frac{2}{3} \times 6 = \frac{2}{3}$ of 6 = 4

iii) $10 \times \frac{3}{5}$ and $\frac{3}{5} \times 10$ $= \frac{30}{5} = 6$

iv) $12 \times \frac{5}{6}$ and $\frac{5}{6} \times 12$ $\frac{60}{6} = 10$

B. Does changing the order we multiply in affect the answer? No

INVESTIGATION 2 ▶ The fractions $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent. Does multiplying by $\frac{2}{6}$ result in the same answer as multiplying by $\frac{1}{3}$?

A. Multiply these numbers by both $\frac{1}{3}$ and $\frac{2}{6}$. Reduce your answer to lowest terms.

i) $4 \times \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}$ $4 \times \frac{2}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

ii) $11 \times \frac{1}{3} = \frac{11}{3} = 3\frac{2}{3}$ $11 \times \frac{2}{6} = \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$

B. Does multiplying by $\frac{2}{6}$ result in the same answer as multiplying by $\frac{1}{3}$? Yes